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GENERAL CRITERION FOR RATING HEAT-EXCHANGER PERFORMANCE

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NOMENCLATURE

A,	combined parameter, equation (16);
c_p,	specific heat at constant pressure;
f,	friction factor;
G,	mass velocity;
h,	specific enthalpy;
h_{av},	average heat-transfer coefficient;
J,	duty parameter, equation (13);
$\dot{m}$,	mass flow rate;
N_{Re},	Reynolds number;
N_S,	number of entropy production units;
$N_{S,\Delta P}$,	$= N_S$ due to friction pressure drop ΔP;
$N_{S,\Delta T}$,	$= N_S$ due to heat transfer across a finite ΔT;
N_{St},	Stanton number;
NTU,	number of heat-transfer units [5];
p,	wetted parameter;
P,	pressure;
q',	heat-transfer rate per unit length;
r_h,	hydraulic radius;
R,	dimensionless ratio of heat-transfer coefficient to pumping power;
s,	specific entropy;
$\dot{S}$,	rate of entropy production [W/K];
T,	absolute temperature;
x,	passage axial coordinate.

Greek symbols

Δ,	increment;
μ,	viscosity;
ρ,	density.

1. INTRODUCTION

A HEAT exchanger is characterized by two types of losses. First, there are losses associated with the heat transfer across the fluid-to-fluid temperature difference, ΔT. These losses are due to a finite exchanger heat-transfer area and, generally, they can be reduced by increasing the heat-transfer area. In addition to the ΔT losses, a heat exchanger is plagued by frictional pressure drops ΔP in its channels. These losses increase with increasing the heat-transfer area. Thus, the ΔT and ΔP losses are said to be coupled in the sense that any design change aimed at reducing one type of loss is likely to have an opposite effect on the other. Due to this coupling, it is often difficult to determine *a priori* whether a proposed design modification will yield a net improvement in heat-exchanger performance.

Recognizing that the ΔT losses and the ΔP losses are both signs of heat-exchanger non-ideality (irreversibility), it is more convenient to evaluate the heat-exchanger losses in terms of one single quantity, namely, its rate of irreversibility or rate of entropy production. It can be shown [1] that this quantity is directly related to the total useful power lost as a result of heat-exchanger non-idealities. Consequently, the impact of a proposed design modification is assessed directly in terms of the change induced in the lost useful power, completely bypassing the issue of conflicting changes induced in the heat-exchanger ΔT and ΔP's.

The use of irreversibility as a general criterion for estimating and minimizing the usable energy wasted in various thermal systems is gaining increasing acceptance [2]. In heat-exchanger design, this concept was first employed by McClintock [3] who reported explicit equations for the local optimum design of fluid passages for either side of a heat exchanger. In spite of this early contribution, the literature concerning the use of irreversibility in heat-exchanger design is scarce. Recently, this author presented an irreversibility analysis and optimum-design method for balanced and imbalanced counterflow heat exchangers [4].

The purpose of this work is to examine the coupling between losses due to heat transfer across the stream-to-stream ΔT and losses caused by fluid friction using the concept of heat-exchanger irreversibility. Based on this concept, the paper proposed the use of a NUMBER OF ENTROPY PRODUCTION UNITS N_S as a basic parameter in describing heat-exchanger performance. This dimensionless group is defined as the entropy production rate or irreversibility rate present in a heat-exchanger passage divided by the stream-to-stream heat-transfer rate to the passage. In these terms, $N_S \to 0$ implies a nearly ideal heat-exchanger passage, one in which the ΔT and ΔP losses together approach zero. Conversely, a large N_S implies a dissipative passage in which the losses are due to excessive stream-to-stream ΔT or frictional ΔP, or both.

To illustrate its use and generality, the N_S criterion is presented *vis-a-vis* a commonly used indicator for heat-exchanger improvement, namely the ratio of heat-transfer coefficient to lost fluid pumping power. It is shown in what follows that increasing the ratio of heat-transfer coefficient to pumping power is not sufficient for claiming improvements in heat-exchanger performance.

2. ENTROPY PRODUCTION IN HEAT EXCHANGERS WITH PRESCRIBED HEAT FLUX DISTRIBUTION

Consider a short heat-exchanger passage of length dx shown in Fig. 1. The fluid mass flow rate $\dot{m}$ in the channel and the heat-transfer rate per unit length q' are specified. The difference between the wall temperature and the bulk temperature of the fluid, ΔT, is not specified but is assumed constant over the length element.

The simple model of Fig. 1, also used in [1], is a good description of single fluid heat exchangers with prescribed heat flux distribution, such as the core of a nuclear reactor or an electric cable cooled by axial forced convection. In two-fluid applications such as counterflow heat exchangers, $\dot{m}$, q' and ΔT depend on each other and on the end states of the two fluids entering the heat exchanger. This interdependence is

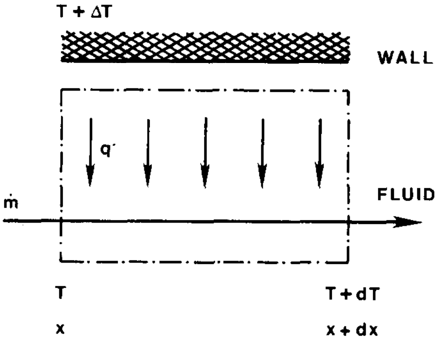

FIG. 1. Schematic diagram of heat-transfer process in a single fluid heat exchanger with prescribed heat flux distribution.

expressed by the familiar effectiveness–NTU relations [5] and can be taken into account in a more refined analysis, such as the one reported by Bejan [4]. The single-fluid model of Fig. 1 is attractive due to its simplicity and is adequate for the task of introducing the concept of number of entropy production units.

We first derive the rate of entropy production in the heat-exchanger passage of Fig. 1. We will perform the derivation in the most general case where the coolant is a pure substance; ideal gases and incompressible liquids are special cases of the more general case derived below. For the control volume shown in the sketch, the net entropy production rate is

$$\mathrm{d}\dot{S} = \dot{m}\,\mathrm{d}s - \frac{q'\,\mathrm{d}x}{T+\Delta T}. \tag{1}$$

Using the first law of thermodynamics on a rate basis,

$$q'\,\mathrm{d}x = \dot{m}\,\mathrm{d}h, \tag{2}$$

allows expression (1) to be written in the following form

$$\mathrm{d}\dot{S} = \dot{m}\left(\mathrm{d}s - \frac{\mathrm{d}h}{T}\right) + \frac{\Delta T}{T^2}\,\frac{\dot{m}\,\mathrm{d}h}{1+\dfrac{\Delta T}{T}}. \tag{3}$$

Recalling that for any pure substance [6]

$$\mathrm{d}h = T\,\mathrm{d}s + \frac{1}{\rho}\mathrm{d}P, \tag{4}$$

the rate of entropy generation per unit length then becomes

$$\frac{\mathrm{d}\dot{S}}{\mathrm{d}x} = \frac{\dot{m}}{\rho T}\left(-\frac{\mathrm{d}P}{\mathrm{d}x}\right) + \frac{\Delta T}{T^2}\,\frac{q'}{1+\dfrac{\Delta T}{T}}. \tag{5}$$

The first term in expression (5) represents the entropy production contribution due to fluid friction in the fluid duct. The second term is the contribution due to heat transfer across the wall–fluid temperature difference. As will be shown, the two contributions are strongly interrelated through the geometric characteristics of the heat exchanger: the hydraulic radius (r_h) and the wetted perimeter (p). And, due to the simplicity of our model, the total rate of entropy generation (5) will always be a function of only two independent parameters corresponding to r_h and p or combinations of r_h and p.

The objective of this analysis is to express the heat-exchanger irreversibility in terms of those parameters which will change as a result of a proposed change in design. The total rate of entropy production can be put in non-dimensional form by normalizing equation (5) with respect to the rate of heat transfer through the wall, q'. Thus, one obtains a number of entropy generation units, N_S, defined as

$$N_S = \frac{T}{q'}\frac{\mathrm{d}\dot{S}}{\mathrm{d}x} = \frac{\dot{m}}{\rho q'}\left(-\frac{\mathrm{d}P}{\mathrm{d}x}\right) + \frac{\Delta T}{T}\left(1+\frac{\Delta T}{T}\right)^{-1}. \tag{6}$$

In order to bring out explicitly the relationship between N_S and various passage parameters ($\dot{m}$, q', ΔT, r_h, p, etc.) one has to appropriately combine expression (6) with the definitions of average heat-transfer coefficient, friction factor, Reynolds number, Stanton number and hydraulic radius, namely

$$h_{\mathrm{av}} = \frac{q'}{p\Delta T}, \tag{7}$$

$$-\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{f}{r_h}\frac{G^2}{2\rho}, \tag{8}$$

$$N_{Re} = \frac{4r_h G}{\mu}, \tag{9}$$

$$N_{St} = \frac{h_{\mathrm{av}}}{c_p G}, \tag{10}$$

$$r_h = \frac{\dot{m}}{Gp}. \tag{11}$$

Expressions (6)–(11) are sufficient for estimating the local number of entropy generation units in any channel. The following example illustrates the power and some interesting features of the N_S criterion.

3. THE EFFECT OF WALL-FLUID ΔT ON THE NUMBER OF ENTROPY GENERATION UNITS N_S

Suppose we want to evaluate the net effect of reducing the wall–fluid temperature difference. For this, it is convenient to combine equations (6)–(11) into an expression showing N_S as a function of ΔT. The end result is

$$N_S = \frac{fN_{Re}^2}{(32)N_{St}^3}\left[\frac{q'\mu}{\dot{m}\rho(c_pT)^{3/2}}\right]^2\left(\frac{\Delta T}{T}\right)^{-3} + \frac{\Delta T}{T}\left(1+\frac{\Delta T}{T}\right)^{-1}. \tag{12}$$

In (12), the dimensionless temperature difference $\Delta T/T$ and the Reynolds number act as independent parameters. The third group appearing in (12),

$$J = \frac{q'\mu}{\dot{m}\rho(c_pT)^{3/2}}, \tag{13}$$

is a fixed heat exchanger duty parameter expressing the ratio of wall heat flux to fluid mass flow rate. In conclusion,

$$N_S = \frac{J^2 fN_{Re}^2}{(32)N_{St}^3}\left(\frac{\Delta T}{T}\right)^{-3} + \frac{\Delta T}{T}\left(1+\frac{\Delta T}{T}\right)^{-1}. \tag{14}$$

As before, the first term in expression (14) is the number of entropy production units due to fluid friction, $N_{S,\Delta P}$, and the second is the contribution resulting from the heat transfer across a finite temperature difference, $N_{S,\Delta T}$. It is clear that both contributions are coupled via the dimensionless temperature difference $\Delta T/T$. Consequently, a minimization of the number of entropy generation units requires a $\Delta T/T$ optimization with respect to the sum of the friction and heat-transfer losses.

The number of entropy generation units has the basic form

$$N_S = \frac{A^2}{3}\left(\frac{\Delta T}{T}\right)^{-3} + \frac{\Delta T}{T}\left(1+\frac{\Delta T}{T}\right)^{-1}, \tag{15}$$

where A is a combined parameter which contains the Reynolds number dependence in addition to the duty parameter dependence

$$A = \left(\frac{3}{32}\frac{f}{N_{St}}\right)^{1/2}\frac{N_{Re}}{N_{St}}J. \tag{16}$$

For a given duty parameter J, the combined parameter A is expected to vary as N_{Re}/N_{St}. This is due to the fact that for common heat-exchanger passage geometries the Reynolds analogy between momentum transfer and heat transfer holds to the extent that the group $(f/N_{St})^{1/2}$ may be regarded as constant over the range $10^2 < N_{Re} < 10^5$ [4].

Figure 2 is a three-dimensional logarithmic plot of equation (15). For values A = constant (<1), there exists an optimum wall-fluid ΔT for which the number of entropy generation units is a minimum. Towards lower values of A, $\Delta T \ll T$ and the minimum is described by the simpler relations

$$\left(\frac{\Delta T}{T}\right)_{\mathrm{opt}} = A^{1/2} \tag{17}$$

$$N_S^{\min} = \frac{4}{3}A^{1/2} = \frac{4}{3}\left(\frac{\Delta T}{T}\right)_{\mathrm{opt}}. \tag{18}$$

The minimum is achieved when the proper trade-off between fluid friction losses ($N_{S,\Delta P}$) and heat transfer ΔT losses ($N_{S,\Delta T}$) occurs and is denoted by $\Delta T/T = (\Delta T/T)_{\mathrm{opt}}$.

When $\Delta T/T < (\Delta T/T)_{\mathrm{opt}}$, the heat-transfer ΔT losses are small compared with the fluid friction losses which make up most of N_S. In this region, if the combined parameter A remains constant the irreversibility number N_S increases

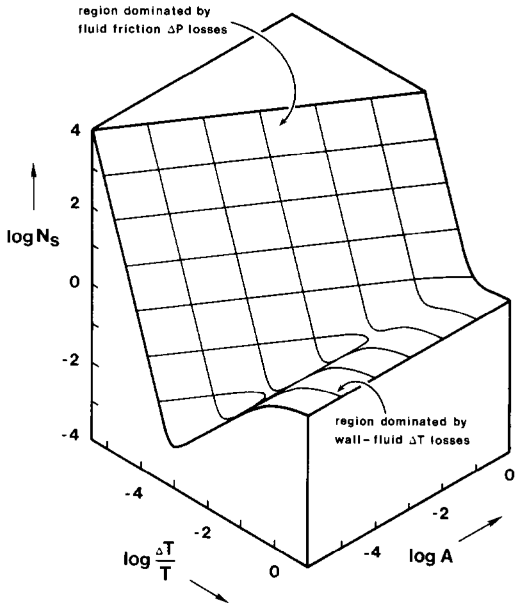

FIG. 2. The local number of entropy production units N_S as a function of the wall-fluid ΔT and the combined parameter A, equation (16).

sharply as $\Delta T/T$ decreases: $N_S \sim (\Delta T/T)^{-3}$. Physically, this dependence is explained as follows. Equations (7), (10), (11) and (13) can be combined to yield

$$\frac{\Delta T}{T} = JN_{St}^{-1}r_h\rho\mu^{-1}(c_pT)^{1/2}. \tag{19}$$

We have seen that, for a given J, the combined parameter A is constant as long as the Reynolds number remains unchanged. Under these conditions, the wall-fluid ΔT varies as the hydraulic radius r_h; as r_h decreases (at constant N_{Re}), the mass velocity and with it the frictional pressure drop and $N_{S,\Delta P}$ increase sharply.

When $\Delta T/T > (\Delta T/T)_{\text{opt}}$, the irreversibility number N_S is dominated by losses due to inadequate thermal contact. In this region, as A is kept constant, N_S increases roughly as $N_S \sim \Delta T/T$. The minimum is thus shallower (less critical) on the $\Delta T/T > (\Delta T/T)_{\text{opt}}$ side of the N_S surface.

To summarize, the route towards minimizing the exchanger local irreversibility consists of choosing the optimum wall-fluid ΔT and at the same time seeking to reduce the value of the combined parameter A.

4. THE RATIO OF HEAT-TRANSFER COEFFICIENT TO PUMPING POWER, AS A CRITERION FOR RATING HEAT-EXCHANGER PERFORMANCE

In many instances, designers choose to maximize the ratio h_{av}/(pumping power) in order to improve the performance of a heat exchanger. As a second example, we will examine here the meaning of maximizing this ratio in view of the number of entropy generation units (N_S) criterion.

A dimensionless group proportional to the ratio of heat-transfer coefficient and pumping power is

$$R = \frac{h_{\text{av}}pT\rho}{\dot{m}}\left(-\frac{\mathrm{d}P}{\mathrm{d}x}\right)^{-1}. \tag{20}$$

Employing the same set of formulae which led to expression (15), R can be put in a form depending only on the two parameters A and $\Delta T/T$,

$$R = 3A^{-2}\left(\frac{\Delta T}{T}\right)^2. \tag{21}$$

Eliminating $\Delta T/T$ between (15) and (21), it is possible to express the local number of entropy generation units in terms

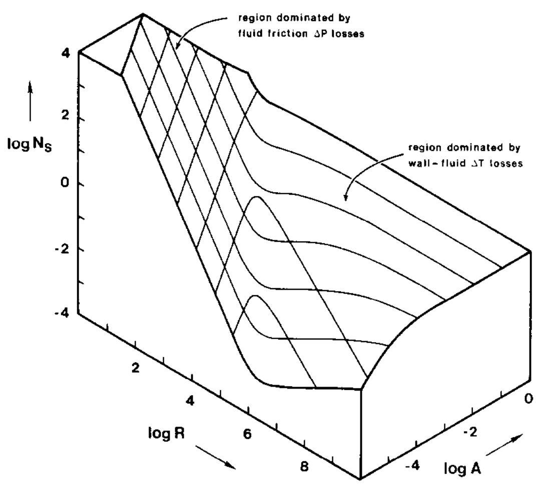

FIG. 3. The local number of entropy production units N_S as a function of A and the ratio of heat-transfer coefficient to fluid pumping power R, equation (20).

of R and A as independent parameters,

$$N_S = \frac{3^{1/2}}{R^{2/3}} + A\left(\frac{R}{3}\right)^{1/2}\left[1 + A\left(\frac{R}{3}\right)^{1/2}\right]^{-1}. \tag{22}$$

Expression (22) is shown plotted as a three-dimensional surface in Fig. 3. The features of this surface are very similar to the features presented in Fig. 2. Thus, for a constant A, there is an optimum ratio R for which N_S is a minimum; for small values of A, the optimum ratio is

$$R_{\text{opt}} = \frac{3}{A}. \tag{23}$$

Based on Fig. 3 and equation (22) we conclude that increasing the ratio of heat-transfer coefficient to pumping power (R) is not sufficient in insuring an improved heat-exchanger performance. Since N_S depends on both R and A, the net effect of a proposed design change can only be evaluated by estimating the changes induced in R and A, and eventually N_S.

For example, if the change in design occurs at constant A (constant N_{Re} and J), the practice of increasing R to achieve better performance applies only in the region $R < R_{\text{opt}}$, i.e. for designs in which the number of entropy generation units is due primarily to fluid friction losses. For designs in which N_S is dominated by losses due to heat transfer across the wall-fluid ΔT (i.e. inadequate thermal contact, $R > R_{\text{opt}}$), maximizing the ratio of heat-transfer coefficient to pumping power is not beneficial (see Fig. 3).

4. CONCLUSION

The number of entropy generation units criterion (N_S) provides a means of evaluating the performance of a heat-exchanger surface directly in terms of the amount of usable energy wasted by the heat exchanger. This criterion is thus closely related to the contemporary trend towards using the concept of second law efficiency in thermal design [7].

To illustrate the use of the N_S criterion we have relied on a simple model representing the class of heat exchangers with prescribed heat flux distribution (Fig. 1). However, formulae similar to equations (6)–(11) can be derived for other classes, such as counterflow heat exchangers as shown by Bejan [4].

The two examples of Figs. 2 and 3 showed that N_S is generally non-monotonic with respect to changing design parameters. Consequently, the use of design rules such as minimizing the wall-fluid ΔT or maximizing the ratio of heat-transfer coefficient to fluid pumping power is not sufficient for seeking improved thermal performance. The N_S criterion is a more adequate measure of thermodynamic imperfection and provides a more complete picture of how various design variables influence the thermal performance.

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A SIMPLE METHOD FOR TIME OF DRYING PREDICTION UNDER CONSTANT WET-BULB TEMPERATURE CONDITIONS

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NOMENCLATURE

A,	drying surface [m^2];
B,	dimensionless driving force as defined by equation (5);
G_i,	flow rate of a dry gas [kg/s];
k_Y,	mass-transfer coefficient as defined by equation (2) [kg/m^{2}s];
k_Y^0,	limiting mass-transfer coefficient at low humidity potentials [kg/m^{2}s];
Le,	Lewis number [dimensionless];
li(u),	logarithmic integral;
N_v,	evaporation flux [kg/m^{2}s];
Pr^0,	limiting Prandtl number for infinite dilution of vapour in a dry gas [dimensionless];
R_s,	$= V_s/A$, specific volume of a solid [m];
S_c^0,	limiting Schmidt number for infinite dilution of vapour in a dry gas [dimensionless];
T^*,	wet-bulb temperature [K];
V_s,	volume of a solid [m^3];
Y^*,	saturated humidity mixing ratio at wet-bulb temperature [kg$_v$/kg$_i$];
Y,	humidity mixing ratio in bulk gas [kg$_v$/kg$_i$].

Greek symbols

α^0,	convective heat-transfer coefficient [W/m^{2}K];
β,	humidity level factor as defined by equation (6) [dimensionless];
γ,	= 1.781072481, Euler's constant;
θ,	time of drying [s];
π,	humidity potential as defined by equation (7) [dimensionless].

Subscripts

1,	refers to entrance state;
2,	refers to exit state;
v,	refers to water vapour;
i,	refers to dry gas.

PROBLEM FORMULATION

FOR TIME of drying prediction in convective dryers with parallel flow of gas and material under constant wet-bulb temperature conditions, the well-known expression: (see e.g. [1, p. 619])

$$\theta = \frac{G_i R_s}{V_s k_Y} \ln\left(\frac{Y^* - Y_1}{Y^* - Y_2}\right) \tag{1}$$

is most frequently used.

The relation (1) is derived from the simple macroscopic mass balance equation, while the drying rate is defined as:

$$N_v = k_Y(Y^* - Y) = k_Y \pi. \tag{2}$$

In deriving (1) it has been assumed that the mass-transfer coefficient k_Y (2) does not depend on absolute humidity nor on humidity potential. Therefore, its value was treated as constant, under the defined flow conditions.

It can be emphasized that from the experiments (see e.g. [2]) it is quite evident that the coefficient k_Y directly depends both on humidity potential and on humidity level. Therefore, by deriving equation (1) the mass-transfer coefficient k_Y (2) cannot in general be considered to be constant.

Considering the fact that the water vapour evaporation is associated with a vapour diffusion process through an essentially still gas, the evaporation flux can be expressed by:

$$N_v = k_Y^0 \ln\left(\frac{1 + Y^*}{1 + Y}\right). \tag{3}$$

Equation (3) should be conceived as an expression defining the mass-transfer coefficient k_Y^0. We can recognize Stefan's, Colburn–Drew's [3] or Spalding's ([4], p. 193) diffusion model in it. In each of the mentioned hypotheses the mass-transfer coefficient k_Y^0 (3) does not depend on humidity potential nor on humidity level, but it is a function of flow conditions and surface geometry. The approach to drying rate analysis similar to (3), is accepted, and the conclusions connected with constancy of k_Y^0 are widely used in the book of Krischer ([5], pp. 248–254). As can be established by the analysis of newer experimental investigations on the evaporation of pure liquids under a wide range of the driving force (see e.g. [6]) the mass-transfer coefficient k_Y^0 is not quite constant. When the driving force changes the value of k_Y^0, however, changes less than that of k_Y.

The relation between the mass-transfer coefficients k_Y and